A MINICOURSE "TOPICS IN THE CALCULUS OF VARIATIONS"

MARTA LEWICKA

This ten-hour compact course, delivered by dr. Marta Lewicka, will provide an introduction to a variety of modern techniques and results in the fields of Calculus of Variations and Analysis, with the eye on applications centered around the mathematical theory of Elasticity and the prestrain-driven Morphogenesis.

The course will be self-contained. A tentative outlay is as follows:

Lectures 1 - 4:

- The notion of Γ-convergence and its fundamental properties.
- Examples of Γ-convergence in the linearized elasticity.
- The fundamental role of Korn's inequality.
- Incompatible nonlinear elasticity: existence of residual stress
- Derivation of the Kirchhoff-like energy as the Γ-limit of the non-Euclidean energies.
- The fundamental role of the geometric rigidity inequality.

Lectures 5 - 6:

- A proof of Korn's inequality.
- A proof of Friesecke-James-Müller's geometric rigidity inequality.

Lectures 7 - 10:

- The hierarchy of the limiting theories for incompatibly prestrained thin plates.
- Energy scaling regimes and the matching properties.
- A proof of the matching property for elliptic strains.
- Some remarks on the regularity of solutions to the Monge-Ampére equations.
- Convex integration for the Monge-Ampére equation.

Prerequisites: Knowledge of Lebesgue measure and integration, Lebesgue spaces L^p and basic knowledge of Sobolev spaces $W^{1,p}$, as well as the standard linear algebra material and the basic differential geometry in $\mathbb{R}^3$ will be assumed.

The lectures will be based on the following papers and preprints:

- O. Oleinik and V. Kondratiev, *On Korn's inequalities*, C.R. Acad. Sci. Paris, **308** Serie I (1989), 483–487.
- G. Friesecke, R. James and S. Müller, *A theorem on geometric rigidity and the derivation of nonlinear plate theory from three dimensional elasticity*, Comm. Pure. Appl. Math., **55** (2002), 1461–1506.
- M. Lewicka and M. Pakzad: *Scaling laws for non-Euclidean plates and the $W^{2,2}$ isometric immersions of Riemannian metrics*, ESAIM: Control, Optimisation and Calculus of Variations, Vol. 17, no 4 (2011), 1158–1173.
- M. Lewicka, M.G. Mora and M. Pakzad: *The matching property of infinitesimal isometries on elliptic surfaces and elasticity of thin shells*, Arch. Rational Mech. Anal. (3) Vol. 200 (2011), 1023–1050.
- M. Lewicka, L. Mahadevan and M. Pakzad: *Models for elastic shells with incompatible strains*, Proceedings of the Royal Society A 470 (May 8 2014), 21–65.
- K. Bhattacharya, M. Lewicka and M. Schaffner: *Plates with incompatible prestrain*, to appear.
- M. Lewicka, L. Mahadevan and M. Pakzad: *The Monge-Ampere constraint: mathcing of isometries, density and regularity, and elastic theories of shallow shells*, to appear.
- M. Lewicka, P. Ochoa and R. Pakzad: *Variational models for prestrained plates with Monge-Ampere constraint*, to appear.

(M. Lewicka) Pittsburgh University, Department of Mathematics 301 Thackeray Hall, Pittsburgh, PA 15260. USA